ENGINEERING MATHS-I
(AM-101, MAY-06)

Note: Section A is compulsory. Attempt any five questions from Section B and C taking at least two questions from each part.

Section-A

1. (a) Find the curvature of curve.

$$y^3 = x^3 + 8$$

at point (1, 3)

(b) If $z = e^{ax+by} f(ax-by)$, find $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y}$

(c) If $u = \sin\left(\frac{x}{y}\right)$ and $x = e^t$, $y = t^{-2}$ find $\frac{du}{dt}$

(d) Find the equation of tangent plane for the surface

$$x^3 + y^3 + 3xyz = 3 \text{ at } \rho(1, 2, -1).$$

(e) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dydx}{1+x^2+y^2}$

(f) Find the centre and the radius of the sphere

$$x^2 + y^2 + z^2 - 6x - 8y - 10z + 1 = 0$$

(g) Test the convergence of the series

$$\sum \frac{n^2+1}{n^3+1}$$

(h) Evaluate $\int_0^2 \int_1^2 \int_0^{yz} xyz\,dxdydz$

(i) If $u = \sin y$ and $x = y \sin x$ then find $\frac{\partial(u,x)}{\partial(x,y)}$.

(j) If $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ prove that $\tanh\frac{u}{2} = \tan\frac{\theta}{2}$

Section-B

2. (a) Trace the curve $y^2 (a + x) = x^2 (a - x)$

(b) Find the curvature of the curve $x^{2/3} + y^{2/3} = a^{2/3}$

3. (a) Prove that the area of loop of the curve $x^3 + y^3 = 3axy$ is $\frac{3a^2}{2}$.

(b) Find the volume of the solid obtained by revolving the curve $y^2(2a-x) = x^3$ about its asymptote.

4. (a). State and prove Euler's theorem.

(b) If $u = \tan^{-1}\left(\frac{y^2}{x}\right)$ prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial y \partial x} + \frac{y^2\partial^2 u}{\partial y^2} = -\sin 2u \sin^2 u$$

5. (a) Expand $x^2y + 3y - 2$ in power of $(x - 1)$ and $(y + 2)$ by using Taylor's series.

(b) Find the maximum and minimum values $x^3 + y^3 - 3axy$ $(a > 0)$.

Section-C

6. (a) Show that the two circles

$x^2 + y^2 + z^2 - y + 2z = 0, \; x - y - 2 = 0$

$x^2 + y^2 + z^2 + x - 3y + z - 5 = 0, \; 2x - y + 4z - 1 = 0$

lie on the same sphere and find its equation.

(b) Find the equation of cone whose vertex is at the origin and which passes through the curve given by the equation

$ax^2 + by^2 + cz^2 = 1$

$x + my + nz = p$

7. (a) Change the order of integration in

$I = \int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$ and hence evaluate.

(b) Prove that

$$\int_1^0 \frac{x\,dx}{\sqrt{1-x^5}} = \frac{1}{2}\beta\left(\frac{2}{5}+\frac{1}{2}\right)$$

8. (a) Test the convergence

$$1+\frac{2x}{2!}+\frac{3^2x^2}{3!}+\frac{4^3x^3}{4!}+\frac{5^4x^4}{5!}+\ldots\ldots\ldots\ldots\infty$$

(b) Verify the series $\sum\frac{4.7\ldots\ldots(3n+1)}{1.2.3\ldots\ldots.n}x^n - 1$ is convergent or divergent.

9. (a) Expand $\cos^8\theta$ in a series of cosines of multiples of θ.

(b) Separate $\tan^{-1}(x + iy)$ into real and imaginary parts.